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**TOP SCHOLAR**



Mana Tohu Mātauranga o Aotearoa  
New Zealand Qualifications Authority

## Scholarship 2024 Physics

Time allowed: Three hours  
Total score: 32

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

**Formulae you may find useful are given on page 3.**

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⚡). This area will be cut off when the booklet is marked.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**



The formulae below may be of use to you.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $v_f = v_i + at$<br>$d = v_i t + \frac{1}{2} at^2$<br>$d = \frac{v_i + v_f}{2} t$<br>$v_f^2 = v_i^2 + 2ad$<br>$F_g = \frac{GMm}{r^2}$<br>$F_c = \frac{mv^2}{r}$<br>$\Delta p = F \Delta t$<br>$\omega = 2\pi f$<br>$d = r\theta$<br>$v = r\omega$<br>$a = r\alpha$<br>$W = Fd$<br>$F_{\text{net}} = ma$<br>$p = mv$<br>$x_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$<br>$\omega = \frac{\Delta\theta}{\Delta t}$<br>$\alpha = \frac{\Delta\omega}{\Delta t}$<br>$L = I\omega$<br>$L = mvr$<br>$\tau = I\alpha$<br>$\tau = Fr$<br>$E_{K(\text{ROT})} = \frac{1}{2} I\omega^2$<br>$E_{K(\text{LIN})} = \frac{1}{2} mv^2$<br>$\Delta E_p = mg\Delta h$<br>$\omega_f = \omega_i + \alpha t$<br>$\omega_f^2 = \omega_i^2 + 2\alpha\theta$<br>$\theta = \frac{(\omega_i + \omega_f)}{2} t$<br>$\theta = \omega_i t + \frac{1}{2} \alpha t^2$ | $T = 2\pi\sqrt{\frac{l}{g}}$<br>$T = 2\pi\sqrt{\frac{m}{k}}$<br>$E_p = \frac{1}{2} ky^2$<br>$F = -ky$<br>$a = -\omega^2 y$<br>$y = A \sin \omega t$<br>$y = A \cos \omega t$<br>$v = A\omega \cos \omega t$<br>$v = -A\omega \sin \omega t$<br>$a = -A\omega^2 \sin \omega t$<br>$a = -A\omega^2 \cos \omega t$<br>$\Delta E = Vq$<br>$P = VI$<br>$V = Ed$<br>$Q = CV$<br>$C_T = C_1 + C_2$<br>$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$<br>$E = \frac{1}{2} QV$<br>$C = \frac{\epsilon_0 \epsilon_r A}{d}$<br>$\tau = RC$<br>$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$<br>$R_T = R_1 + R_2$<br>$V = IR$<br>$F = BIL$<br>$V = BvL$<br>$F = Bqv$<br>$F = Eq$<br>$E = \frac{V}{d}$ | $\phi = BA$<br>$\epsilon = -\frac{\Delta\phi}{\Delta t}$<br>$\epsilon = -L \frac{\Delta I}{\Delta t}$<br>$\frac{N_p}{N_s} = \frac{V_p}{V_s}$<br>$E = \frac{1}{2} LI^2$<br>$\tau = \frac{L}{R}$<br>$I = I_{\text{MAX}} \sin \omega t$<br>$V = V_{\text{MAX}} \sin \omega t$<br>$I_{\text{MAX}} = \sqrt{2} I_{\text{rms}}$<br>$V_{\text{MAX}} = \sqrt{2} V_{\text{rms}}$<br>$X_C = \frac{1}{\omega C}$<br>$X_L = \omega L$<br>$V = IZ$<br>$f_0 = \frac{1}{2\pi\sqrt{LC}}$<br>$v = f\lambda$<br>$f = \frac{1}{T}$<br>$n\lambda = \frac{dx}{L}$<br>$n\lambda = d \sin \theta$<br>$f' = f \frac{V_w}{V_w \pm V_s}$<br>$E = hf$<br>$hf = \phi + E_K$<br>$E = \Delta mc^2$<br>$\frac{1}{\lambda} = R \left( \frac{1}{S^2} - \frac{1}{L^2} \right)$<br>$E_n = -\frac{hcR}{n^2}$ |
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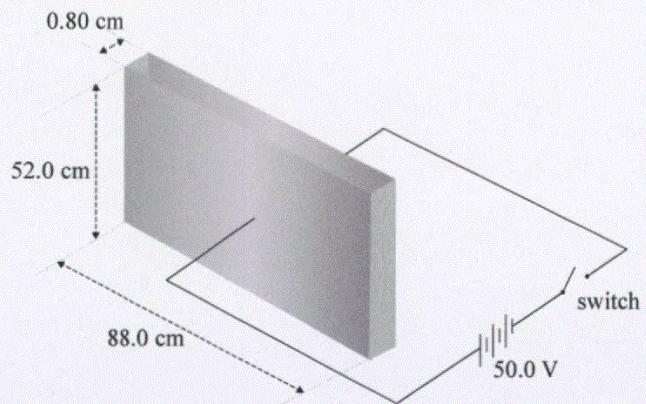


### QUESTION ONE: CAPACITOR TANK

Permittivity of free space =  $8.85 \times 10^{-12} \text{ F m}^{-1}$

Relative permittivity of air = 1.0

Hannah uses a rectangular tank, as shown in the diagram, to carry out a series of experiments investigating the behaviour of a parallel plate capacitor. The front and back of the tank are made of conducting metal plates 88.0 cm wide and 52.0 cm high. The sides and bottom of the tank are 0.80 cm wide, and are made of thin, non-conducting plastic.



- (a) (i) For her first experiment, Hannah closes the switch to connect the plates to the 50.0 V power supply.

Show that the charge stored on the plates is  $2.5 \times 10^{-8} \text{ C}$ .

Explain why this value should be stated to two significant figures.

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 1 \times (0.88 \text{ m} \times 0.52 \text{ m})}{0.8 \times 10^{-2} \text{ m}}$$

$$= 5.0622 \times 10^{-10} \text{ F}$$

$$Q = CV = 5.0622 \times 10^{-10} \times 50 \text{ V}$$

$$= 2.53 \times 10^{-8} = 2.5 \times 10^{-8} \text{ C (2sf)}$$

in the question are measurements provided, the lowest number of significant figures is 2, hence why the answer is rounded as such.

- (ii) Explain why the amount of energy stored in the charged capacitor is only half of the total energy provided by the power supply.

As the capacitor charges, half the energy carried in the electrical current does not make it onto the plates of the capacitor (stores energy in electric field via charge separation) and is returned to the battery or lost as heat to resistance.



- (b) The tank can be filled with a non-conducting oil with a relative permittivity of 2.1. No charge is transferred from the plates to the oil. There is a valve in the bottom of the tank that can be opened to drain the oil from the tank.

For her second experiment, Hannah starts with the tank empty, the plates uncharged, and the switch open. She then carries out the following steps in sequence to achieve a potential difference between the plates that is greater than 50.0 V:

Step 1: Close the switch.

Step 2: Fill the tank with oil.

Step 3: Open the switch.

Step 4: Drain the oil from the tank.

- (i) Explain the effect of each of these steps, and how this sequence results in a potential difference that is greater than 50.0 V between the plates.

① When the switch is closed, the capacitor charges up until it holds the  $2.5 \times 10^{-8} \text{ C}$ , and the potential across it is 50 J/C. (50V)

② Filling the tank with oil acts as putting a dielectric between the plates. The plates of the capacitor can now fit more charge onto the plates, so more flows on from the battery. Since the voltage across the capacitor drops below 50V, it has to re-climb again to reach 50V by putting on more charge.

③ breaks the circuit. The capacitor holds its charge across the electric field \*

④ the capacitance is reduced back to original. more charge than  $2.5 \times 10^{-8} \text{ C}$  is on the plates, so since  $C$  is back to original,  $V = Q/C$ ,  $Q$  has increased so  $V$  has increased, but at 50V.

\* the break <sup>from the switch</sup> is essentially a  $\infty$  resistance, so time constant  $\rightarrow \infty$ .  
 $\therefore$  charge doesn't flow off plates.



- (ii) Calculate the value of the potential difference between the plates that results from following the sequence of steps described on page 5.

$$\text{Capacitance with oil} = 5.0622 \times 10^{-10} \times 2.1 \\ = 1.063 \times 10^{-9} \text{ F}$$

$$Q = 1.063 \times 10^{-9} \times 50 = 5.315 \times 10^{-8} \text{ C}$$

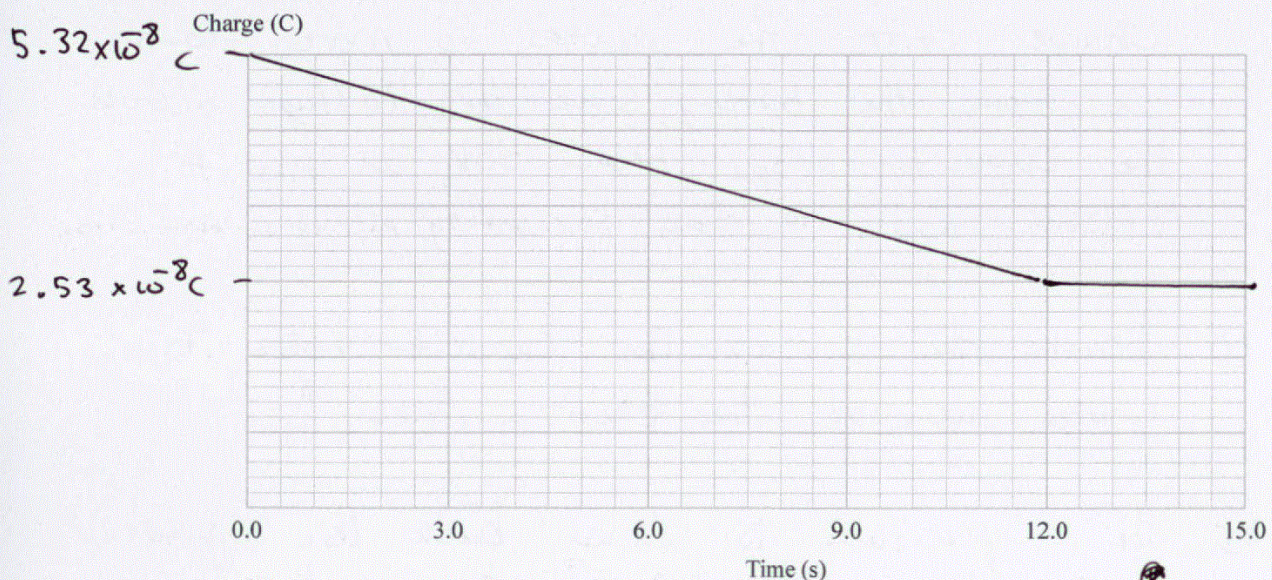
now oil drains, C returns to  $5.0622 \times 10^{-10} \text{ F}$

$$V = Q/C = \frac{5.315 \times 10^{-8}}{5.0622 \times 10^{-10}} = 105 \text{ V}$$

- (c) For her third experiment, Hannah fills the tank with oil and closes the switch so that the potential difference across the plates is 50.0 V. At time  $t = 0.0 \text{ s}$ , she opens the valve and oil drains from the tank while the switch remains closed. The tank is completely empty after 12.0 s. Assume the oil flows from the tank at a constant rate.

Sketch a graph on the axes below to show how the charge stored on the plates changes over the first 15.0 s.

Indicate key values on the charge axis.



V is constant.

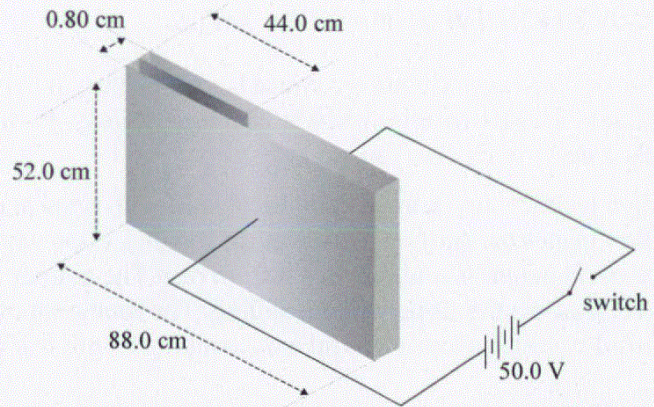
~~Q~~ decreases linearly  
Q decreases linearly

↑  
So no moment  
of  
charge.

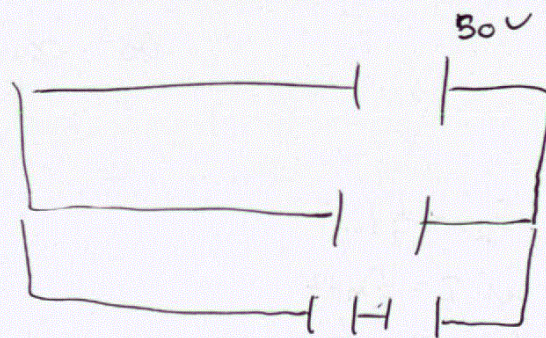


- (d) For her last experiment, Hannah now adds a third parallel thin metal plate between the two original plates. The new plate is the same height as the two original plates, but is only half the width, and has negligible thickness.

Using appropriate physics concepts, explain the effect that adding the third plate will have on the total capacitance of the tank.



Capacitance depends on exposed surface area between the ~~top~~ plates that can be used to store charge, and on electric field. By adding the thin plate, the system is now one plate of half exposed area and 2 plates of half exposed area at half distance <sup>in series, with everything in</sup> parallel with each other (shown below) the two in series have half the area but half the distance as well, so the capacitance of each is unchanged, but the net capacitance is half in this branch. Capacitance is ~~series~~ <sup>parallel</sup> added (because it really is one big plate) so this half capacitance is added to another half capacitance is the ~~bigger~~ other branch (since its area was halved) so the net capacitance is unchanged.



$$C = 5.06$$

$$C = 2.53 \times 10^{-10}$$

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$$C = 2.53 \times 10^{-10}$$

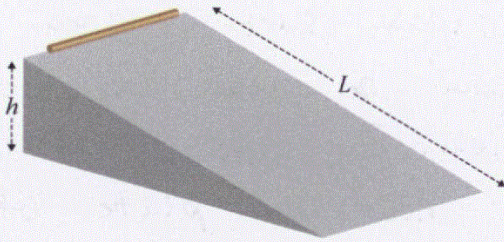
$$C_{TT} = 5.06 \times 10^{-10} \text{ F}$$



## QUESTION TWO: MARTIAL ARTS

Several cultures around the world have a history of martial arts using a staff. Some examples include Japanese Bōjutsu, Portuguese Jogo Do Pau, and Māori Mau Rākau.

Ben is practising using a cylindrical staff with mass  $m$  and radius  $r$ . Ben places the staff on its side at the top of a slope with length  $L$  and vertical height  $h$ , and releases it from rest. The cylindrical staff rolls freely down the slope without sliding. The rotational inertia  $I$  of a solid cylinder about a central axis along its length is  $I = \frac{1}{2}mr^2$ .



Source: [www.festival.nz/article/a-window-into-te-ao-m%C4%81ori-the-taiaha/](http://www.festival.nz/article/a-window-into-te-ao-m%C4%81ori-the-taiaha/)

- (a) Show that the time,  $t$ , taken for the staff to reach the bottom of the slope is given by:

$$t = L\sqrt{\frac{3}{gh}}$$

where  $g$  = the acceleration due to gravity.

Conserve energy, ( $E_p = E_k + E_{kr}$ )

$$mgh = \frac{1}{2}mv_x^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_x^2}{r^2}\right) *$$

$$gh = \frac{3}{4}v_x^2$$

$$\frac{4gh}{3} = v_x^2 \Rightarrow v_x = 2\sqrt{\frac{gh}{3}}$$

$$v_f = \frac{2\sqrt{gh}}{\sqrt{3}} \quad v_i = 0 \quad d = L$$

$$(v_f = v_x)$$

$$t = ?$$

$$d = \frac{v_i + v_f}{2} t$$

$$t = \frac{2d}{v_i + v_f} = \frac{2L}{\frac{2\sqrt{gh}}{\sqrt{3}}}$$

$$t = \frac{2L\sqrt{3}}{2\sqrt{gh}} = L\sqrt{\frac{3}{gh}}$$

as required.

\* Since  $E_{kr} = \frac{1}{2}I\omega^2$   
 and  $I = \frac{1}{2}mr^2$   
 and  $\omega = \frac{v_x}{r}$



(b) By referring to relevant physics principles, discuss the validity of these two statements:

- (i) **Statement 1:** For an object with fixed mass to have linear acceleration, it must experience a non-zero net force.

This statement is a discussion of Newton's law,  $F=ma$ . Acceleration in a linear direction is equal to the net force in that direction, divided by the mass of the object. If the net force is 0, an object can still be moving (e.g. terminal velocity) but cannot be accelerating. Another way to see this is that for an object to accelerate, it must gain kinetic energy, which must come from work provided by a linear force. Therefore in order to accelerate, a non-zero net force in a given direction must be present.

- (ii) **Statement 2:** For an object with fixed mass to have angular acceleration, it must experience a non-zero net torque.

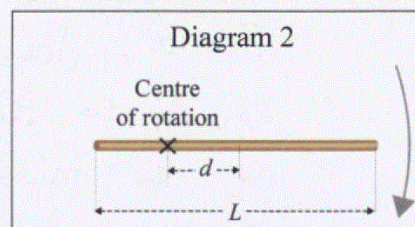
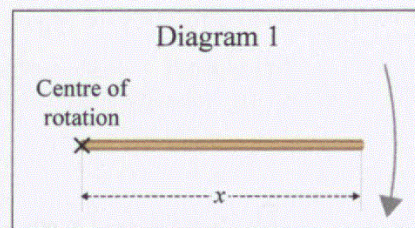
Angular acceleration can be supplied by work. Imagine a man spinning on a chair, holding weights with his hands outstretched. By drawing his hands inward, the system's inertia lowers ( $I \propto mr^2$ ) but the angular velocity,  $\omega$ , must increase, because IN THE ABSENCE OF AN EXTERNAL TORQUE the angular momentum,  $L = I\omega$  must hold constant. To increase  $\omega$  the system must have experienced an angular acceleration. The rotational energy supplied to increase  $\omega$  came from work done by the man's arms, not an external net torque.

(work and torque act orthogonally to each other)



- (c) For a uniform thin rod with total length  $x$  and mass  $m$  rotating about one end, as shown in Diagram 1, the rotational inertia  $I$  is given by  $I = \frac{1}{3}mx^2$ .

While performing a sideways strike, a staff can be modelled as a thin, uniform rod rotating about a vertical axis through the centre of rotation, which is in a fixed position. The centre of rotation is closer to one end of the rod than the other, as shown in Diagram 2. The rotational inertia of a thin, uniform rod rotating about an axis part way along the rod can be found by adding the rotational inertias of the sections of the rod either side of the centre of rotation.



Show that the rotational inertia of a thin, uniform rod of length  $L$  and mass  $m$  rotating about a point a distance  $d$  from the centre of mass is given by:

$$I = m \left( \frac{L^2}{12} + d^2 \right)$$

Take  $I$  of stick to right of centre of rotation  
Mass should be  $\frac{L}{2} + d$  m, as that is the length proportion  
 $x = \frac{L}{2} + d$

$$I_{\text{right}} = \frac{1}{3} \left( \frac{1}{2} + \frac{d}{L} \right) m \left( \frac{L}{2} + d \right)^2 = \frac{1}{3} m \left( \frac{1}{2} + \frac{d}{L} \right) \left( \frac{L^2}{4} + Ld + d^2 \right)$$

$$I_{\text{right}} = \frac{1}{3} m \left( \frac{L^2}{8} + \frac{Ld}{2} + \frac{d^2}{2} + \frac{dL^2}{4L} + \frac{d^2L}{L} + \frac{d^3}{L} \right)$$

$$I_{\text{left}} = \frac{1}{3} m_L x_L^2 \quad m_L = \left( \frac{L}{2} - d \right) m \text{ (as that is the proportion)}$$

$$I_L = \frac{1}{3} \left( \frac{1}{2} - \frac{d}{L} \right) m \left( \frac{L}{2} - d \right)^2 = \frac{1}{3} m \left( \frac{1}{2} - \frac{d}{L} \right) \left( \frac{L^2}{4} - dL + d^2 \right)$$

$$I_{\text{left}} = \frac{1}{3} m \left( \frac{L^2}{8} - \frac{Ld}{2} + \frac{d^2}{2} - \frac{dL^2}{4L} + \frac{d^2L}{L} - \frac{d^3}{L} \right)$$

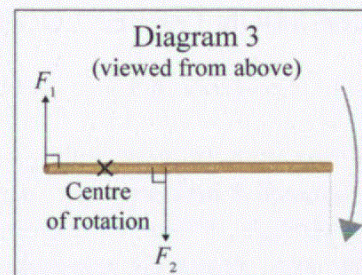
$$I_T = I_R + I_L = \frac{1}{3} m \left( \frac{L^2}{8} + \frac{L^2}{8} + \frac{d^2}{2} + \frac{d^2}{2} + 2d^2 \right)$$

$$I_T = \frac{1}{3} m \left( \frac{L^2}{4} + 3d^2 \right) = \frac{1}{12} m \left( \frac{L^2}{12} + d^2 \right)$$

as required.



- (d) Ben makes the staff rotate around a vertical axis by applying two horizontal forces  $F_1$  and  $F_2$ , one with each hand. These forces push the ends of the staff in opposite directions, as shown in Diagram 3 (viewed from above). The forces  $F_1$  and  $F_2$  produce a net torque that gives the staff an angular acceleration about the centre of rotation. The forces  $F_1$  and  $F_2$  are spaced equally from the centre of rotation and are applied perpendicular to the staff's length. The centre of rotation stays in the same position.



- (i) Discuss the relative sizes of the forces  $F_1$  and  $F_2$  needed to cause the staff to have an angular acceleration while the centre of rotation does not move.

~~$F_2$  needs to carry the inertia of the RHs of the rod with it, which is clearly bigger than that of the LHS, so  $F_2$  needs to be of greater magnitude than  $F_1$ .~~

back page.

- (ii) A sideways strike can be blocked by holding a second staff vertically in front of the rotating staff.

Explain the position and direction that the blocking force should be applied to stop the rotating staff most effectively.

Explain the effect this blocking force has on the motion of the rotating staff.

Source: [www.reddit.com/r/martialarts/comments/13cdmpy/mau\\_rakau\\_and\\_wharetutaua/?rdt=43199](https://www.reddit.com/r/martialarts/comments/13cdmpy/mau_rakau_and_wharetutaua/?rdt=43199)

The blocking force should apply a torque against the one being generated by  $F_1$  or  $F_2$ . The force should be  $\perp$  (orthogonal) to the motion of the end of the staff, as far from the axis of rotation <sup>of the swinging staff</sup> to maximise torque, and block the staff. This torque decelerates the swinging staff to a stop, provided it is larger than the other applied torque.



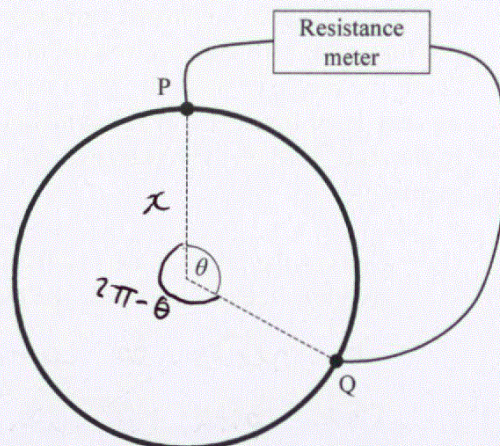
### QUESTION THREE: LOOPS AND RINGS

Acceleration due to gravity =  $9.81 \text{ m s}^{-2}$

A long, straight piece of thin resistive wire has a total resistance  $R$  between one end and the other. The piece of wire is now bent into a circular shape and the ends are connected together to make a single circular loop.

A resistance meter is connected between two points on the loop, P and Q, which are separated by an angle  $\theta$  around the loop as shown.

Point P remains fixed at the top of the loop, but point Q can be moved to any point around the edge of the loop.



- (a) By considering the resistance of the two sections of the circular loop (clockwise from P to Q, and anticlockwise from P to Q), show that  $R_{PQ}$ , the total resistance of the loop between the points P and Q, is given by the expression:

$$R_{PQ} = \frac{R\theta}{2\pi} \left( 1 - \frac{\theta}{2\pi} \right)$$

take radius of loop to be  $x$

then P to Q clockwise is  $x\theta$ .

So  $\frac{x\theta}{2\pi x}$  is the proportion of the circle  $\Rightarrow \frac{\theta}{2\pi}$

~~the resistance of this section should be  $R \left( \frac{\theta}{2\pi} \right)$~~

anti-clockwise, the proportion is  $\frac{(2\pi - \theta)x}{2\pi x} = \frac{2\pi - \theta}{2\pi}$

the proportions of the loop should be multiplied together, then by the resistance of the entire loop to achieve the desired resistance

$$\left( \frac{\theta}{2\pi} \right) \left( \frac{2\pi - \theta}{2\pi} \right) R = R \left( \frac{2\pi\theta - \theta^2}{4\pi^2} \right) = \frac{R\theta}{2\pi} \left( 1 - \frac{\theta}{2\pi} \right)$$

$$\left[ \frac{\theta}{2\pi} \times \left( 1 - \frac{\theta}{2\pi} \right) \times R \right] = \frac{R\theta}{2\pi} \left( 1 - \frac{\theta}{2\pi} \right)$$



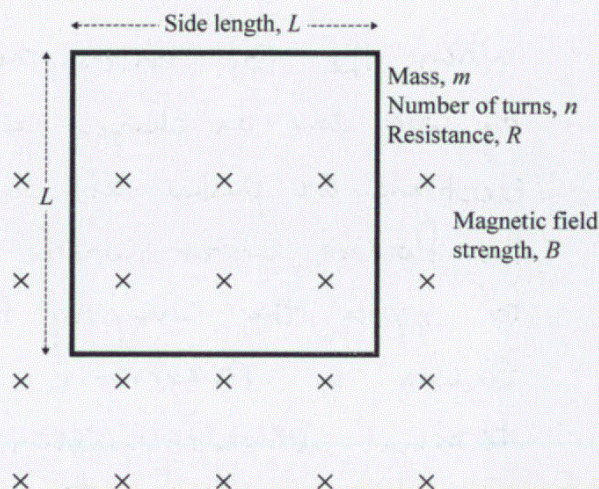
- (b) Explain, using physics concepts, where point Q should be placed to give the largest value of  $R_{PQ}$ , and determine the maximum value of  $R_{PQ}$  for a length of wire with resistance  $R = 10 \Omega$ .

Q should be directly below P so  $\theta = \pi$ .

By doing so, arc length travelled is maximised and hence the effective resistance is maximum

$$\text{max} = \frac{10\pi}{2\pi} \left(1 - \frac{1}{2}\right) = 5/2 = 2.5 \Omega$$

- (c) A square coil of wire with mass,  $m$ , resistance,  $R$ , and number of turns,  $n$ , falls freely into a uniform magnetic field as shown. The effects of air resistance can be ignored.



- (i) By considering the forces acting on the coil, show that the terminal velocity of the coil as it falls into the field is given by:

$$v = \frac{mgR}{B^2 L^2 n^2}$$

terminal velocity is achieved when the downward gravitational force is equal to the upward magnetic force.

the magnetic force on each turn  $\Rightarrow BIL$   
 so  $\sum F = nBIL$

the Voltage on each  $\Rightarrow BLv$  so  $V_{\text{net}} = nBLv$   
 $V = IR \Rightarrow I = \frac{nBLv}{R}$

$$F = \frac{n^2 B^2 L^2 v}{R} \quad [nBIL]$$

$$F_g = F_m \Rightarrow mg = \frac{n^2 B^2 L^2 v}{R}$$

$$v = \frac{mgR}{B^2 L^2 n^2}$$



- (ii) By choosing realistic values for  $m$ ,  $n$ , and  $L$ , estimate the terminal velocity of a small coil with resistance  $R = 1.0 \Omega$  as it falls into a magnetic field of strength  $B = 1.0 \times 10^{-3} \text{ T}$ .

Discuss the motion of your coil as it enters the field.

lets say, <sup>60</sup>200 windings, mass of <sup>0.02 kg</sup>200g (0.2 kg)  
and <sup>1m</sup>30cm side-length <sup>1m</sup>60cm

$$V = \frac{0.02 \times 9.81 \times 1}{200 \times 0.3^2 \times (10^{-3})^2} = \frac{0.02 \times 9.81 \times 1}{60^2 \times 1 \times 10^{-6}} = 54.5 \text{ ms}^{-1}$$

When the coil enters the field, a voltage is induced by work done on charges across <sup>the</sup> side  $L$  to the motion. electrons are pushed left, so a current flows anticlockwise. the flowing current creates a magnetic field which acts to oppose the changing flux through loop. This action causes a magnetic force upward. The loop reaches terminal velocity when the induced voltage is large enough to generate a current, ~~and~~ which creates force which opposes gravity. The coil gradually accelerates (negative jerk  $\frac{da}{dt} < 0$ ) until this state is reached, as no further acceleration occurs ( $\frac{da}{dt} = 0$ ). If the whole loop makes it into the field, the flux no longer changes so the force disappears, opposing induced voltages occur and no current flows. This causes the coil to accelerate from gravity beyond terminal velocity. ( $54.5 \text{ ms}^{-1}$ )



- (d) A square coil is unwound, and the same piece of wire is used to make a second square coil with sides twice as long as the original coil.

Use physics concepts to explain how the terminal velocity of the new coil compares to that of the initial coil when dropped into the same magnetic field.

the effective length of the unwound piece is  $4 \times L \times n = 4Ln$  m to make a square of sides  $2L$ , its effective length would be  $8L \times (\text{new number})$   
 $8Lx = 4Ln \quad x = \frac{1}{2}n$

the number of windings is therefore halved. The induced Voltage expression is not changed, since  $L$  being doubled is offset by  $n$  being halved.

The Force (magnetic) expression is unchanged since the Length is doubled but  $n$  is halved.

this leads to an equivalent terminal velocity, when dropped, since the resistance of the coil is still  $1\Omega$ .

This makes sense  $\Rightarrow$  the same length of total wire is falling, so the terminal velocity should be unaltered.



### QUESTION FOUR: LASER COOLING

Planck's constant =  $6.63 \times 10^{-34} \text{ J s}$

Mass of a sodium atom =  $3.82 \times 10^{-26} \text{ kg}$

Speed of light =  $3.00 \times 10^8 \text{ m s}^{-1}$

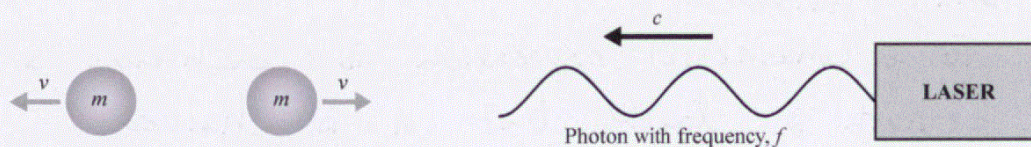
Physicists use lasers to transfer momentum and slow down gaseous atoms. This can cool a gas down to temperatures close to absolute zero.

The momentum of a photon is related to its wavelength by the relationship:

$$p = \frac{h}{\lambda}$$

A laser of frequency,  $f$ , is shone into a container holding gaseous sodium atoms. The energy of each photon is slightly less than the difference in energy between two electron energy levels in a sodium atom.

Gaseous sodium atoms can be moving in any direction, but for the purposes of Question Four, we will only consider one-dimensional motion – either towards or away from the laser.



- (a) Explain which sodium atom, the one moving towards the laser or the one moving away from the laser, can absorb a photon from the laser.

In order to absorb a photon, the energy of the photon  $[E = (hf)]$  must be equal to the energy change of a electron jumping between energy levels. The energy of emitted photons is slightly less than the required difference in energy, so the energy of the photons needs to be increased. By moving toward the laser, a sodium atom can encounter more photons per second, hence increasing their frequency, so that the energy is slightly increase to the required level. Since  $E = hf$ .  $\therefore$  the sodium atom moving toward the laser can absorb a photon via a doppler shift, which gives the correct, slightly higher frequency, to match the required quantised energy change.



- (b) Explain how the momentum of each of the sodium atoms, the one moving towards the laser and the one moving away from the laser, may be affected by the photon.

The one moving toward it <sup>to the right</sup> can absorb the photon <sup>which is moving left</sup>. By doing so, the reduction momentum of this photon is imparted to the sodium atom, so to conserve momentum the atom slows down. So that final momentum to the right = initial momentum to the right.

The one moving away does not interact. There is not a difference in electron energy levels that aligns with the Doppler shifted photon (relative to the left atom) so no photon-absorption can take place, since electrons only occupy discrete, stable electron orbits, and cannot exist between them.  $\therefore$  the left moving one is unaffected.

- (c) For an observer moving at speed  $v$ , where  $v$  is much less than the speed of light, the apparent change in frequency  $\Delta f$  of an electromagnetic wave of frequency  $f$ , due to the Doppler effect, can be approximated by:

$$\Delta f = \frac{fv}{c}$$

A transition between energy levels in a sodium atom occurs when a photon with wavelength  $5.89 \times 10^{-7} \text{ m}$  is absorbed. The average speed of a gaseous sodium atom at room temperature is  $5.70 \times 10^2 \text{ m s}^{-1}$ .

Assume atom moves toward photon

Calculate the percentage change in momentum of a sodium atom caused by the absorption of a photon with wavelength  $5.89 \times 10^{-7} \text{ m}$ .

$$p_{\text{photon}} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.89 \times 10^{-7}} = 1.2156 \times 10^{-27} \text{ J s m}^{-1}$$

$$c = f\lambda \quad f = \frac{c}{\lambda} \quad \Rightarrow \Delta f = \frac{v}{\lambda} = \frac{5.7 \times 10^2}{5.89 \times 10^{-7}} = 9.68 \times 10^{12}$$

the apparent frequency =  $\Delta f + f = 5.093 \times 10^{14} \text{ Hz}$

$$p_0 = (5.7 \times 10^2)(3.82 \times 10^{-26}) - 1.2156 \times 10^{-27} = 2.177278 \times 10^{-23} \text{ kg m s}^{-1}$$

$$p_a = \frac{v}{\lambda}(5.89 \times 10^{-7}) = 2.177278 \times 10^{-23} \text{ kg m s}^{-1}$$

$$v_x = 570$$

$$\frac{2.177278}{2.1774} = 0.9999 \quad \text{so } 0.0056 \% \text{ reduction}$$

Question Four continues on the following page.



- (d) Despite being used for slowing atoms down, this process can sometimes result in an atom moving faster than it was originally. After absorbing a photon, the sodium atom can then emit a photon when the electron drops back down to its original energy level. The emitted photon can be emitted in any direction.

Explain how this process can result in the atom moving faster than its original speed if the photon is emitted in the same direction the original photon was travelling (away from the laser).

When the atom absorbs the photon, it ~~has~~ <sup>slows</sup> down, but the apparent frequency of the photon from the Sodium's perspective is greater than the actual apparent frequency of the photon. The apparent wavelength is therefore shorter. The momentum of the original photon,  $\frac{h}{\lambda}$  uses the true wavelength in the formula. As such, when a photon is re-emitted by Sodium, the energy change produces a photon of higher frequency than the original photon. The new photon has  $\therefore$  a lower wavelength and hence a higher momentum than the original.  $\therefore$  Although the Sodium atom is slowed by the initial momentum change, the final momentum change is higher, so the Sodium atom moves faster along its original trajectory.



Extra space if required.  
Write the question number(s) if applicable.

QUESTION  
NUMBER

2)

d) i)

the Rhs has a higher inertia so  $F_2$  must be bigger than  $F_1$  if both are to supply the same angular acceleration around the centre of rotation.